SCIENCE AND TECHNOLOGY – MAGIC SQUARES

1. MAGIC SQUARES: BASIC DEFINITIONS

A – NUMERIC MAGIC SQUARES

1.01 **Cell** – Each one of $n^2 = n \times n$ small squares that constitute the *n Rows* and *n Columns* of the square. *Cells* can be filled with **Numbers**, **Letters/Characters** or **Geometric Shapes/Pieces**.

1.02 **Closed Magic Line** – Lines that connect the centers of *Cells* in numeric sequence, including the return to the first, e.g. 1,2,3,…,n^2,1 for a *Normal Magic Square* (after Claude Bragdon).
When the areas between the *Magic Line* are filled with different colours, some very interesting patterns may be created, called **Sequence Patterns** (after Jim Moran).

1.03 **Column** – Set of *n* vertical *Cells*. A square of *Order n* has *n Columns*.

1.04 **Complementary Pair** or **Complementary Numbers** – Pair of *Cells* or Pair of Numbers which sum is equal to the sum of the first and last terms of the series, e.g. $1+n^2$ for a *Normal Magic Square*.

1.05 **Diagonals**

1.05.1 **Bent Diagonal** – Diagonal, with the same *n* number of *Cells* as the *Order n* of the Square, that:

- Starts at any edge *Cell* of the Square and finish at the opposite edge *Cell*;
- Is composed by two V-shape perpendicular and symmetric segments with the vertice coinciding with: the center of the Square, for *Even Order*; the center of the central *Cell*, for *Odd Order*.

There are 4x*n Bent Diagonals*, distributed as follows:

- *Even Order*: Continuous, 2(*n*+2), *Wrap-Around*, 2(*n*-2);
- *Odd Order*: Continuous, 2(*n*+1), *Wrap-Around*, 2(*n*-1).

Bent Diagonals also sum the *Magic Sum S* and are the main characteristic of **Benjamin Franklin** "*Magic Squares*".

1.05.2 **Broken Diagonal** – Two *Short Diagonals* – with the same or different number of *Cells* – that are parallel to a *Main Diagonal*, but on opposite sides, and when connected, contain the same number of Cells, as each *Row*, *Column* or *Main Diagonal*, i.e. the *Order* n of the *Magic Square*.

Broken Diagonals also sum the *Magic Sum S* and are the main characteristic of the *Pandiagonal Magic Squares*.

1.05.3 **Main Diagonal** – Each one of the two diagonals, called **Leading** or **Left Diagonal** and **Right Diagonal**, constituted with *n Cells*, which connect the opposite corners of the square.

1.05.4 **Opposite Short Diagonals Pair** – Two *Short Diagonals* which are parallel and on opposite sides of a *Main Diagonal* and contain the same number of *Cells*.

1.05.5 **Pandiagonal** – That include all diagonals*: Main Diagonals* and *Broken Diagonals.*

1.05.6 **Short Diagonal** – Diagonal that is parallel to one of the *Main Diagonals* and intercept two adjacent sides of the square. A *Short Diagonal* may have 1 to n-1 *Cells*.

1.05.7 **Wrap-Around Diagonal** – *Bent Diagonal* obtained through wrap-around process.

1.06 **Magic Sum** or **Magic Number** or **Magic Constant** – The constant sum of the *n Cells* of each *Row*, *Column*, *Main Diagonal*, etc.. For a *Normal Magic Square* the *Magic Sum S* can be calculated by the formula $\mathbf{S=}\boldsymbol{n}\mathbf{(}\boldsymbol{n}^{\mathbf{2}}\mathbf{+1)/2}$, where *n* is the *Order of the Magic Square*.

1.07 **Normal** or **Pure** or **Traditional Magic Square** of **Order n** – *Magic Square* where the numbers used for filling the n^2 *Cells* are consecutive positive integers from 1 to n^2.

1.08 **Order n** – There are four classes of *Magic Squares* according the *Order*:

1.08.1 **Even** – When *n* is even, i.e. *n*=2,4,6,8,…;

1.08.2 **Odd** – When *n* is odd, i.e. *n*=3, 5, 7,…;

1.08.3 **Doubly–Even** – When *n* is multiple of 4, i.e. *n*=4, 8, 12,…;

1.08.4 **Singly** or **Oddly–Even** – When *n* is even but is not divisible by 4, i.e. *n*=6, 10, 14,….

1.09 **Row** – Set of *n* horizontal *Cells.* A square of *Order n* has *n Rows*.

1.10 **Square of Order n** – Square composed with *n Rows*, *n Columns* and n^2 *Cells*.

1.11 **Symmetrical Pair** or **Symmetrical Cells** – Pair of *Cells* diametrically equidistant or symmetric in relation to the Center of the Square.

1.12 **Wrap-Around** – To connect the square opposite sides (i.e. left-right or up-down) in order to make a continuous cylindrical surface where the opposite sides overlay.

B – LETTERS/WORDS/CHARACTERS MAGIC SQUARES

1.13 **Palindrome** – A number, word or phrase that can be read the same in different directions (e.g. right-to-left or left-to-right). However, as it is the case for the *Su Hui Xuan Ji Tu Palindrome* poem, the verses, in Chinese language, can also be read in different directions, not with the same meaning, but yet meaningful.

C – GEOMETRIC MAGIC SQUARES

1.14 **Geometric Magic Square** or **Geomagic Square** An array of n^2 *Cells* (*n Rows* x *n Columns*) each occupied by a distinct geometrical (usually planar) **Shape** or **Piece**, such that *n* of them taken from any *Row*, *Column* or *Main Diagonal* can be assembled to create a **Larger Constant Shape** or **Piece** known as the *Magic Target*. By the **Dimension** *of a Geomagic Square* is meant the dimension of its *Shapes* or *Pieces*.

1.15 **Magic Target** – *The Larger Constant Shape* or *Piece* formed by the union of the *n Shapes* or *Pieces* occurring in any *Row*, *Column*, or *Main Diagonal*. The *Shapes* or *Pieces* used are usually planar, but may be of any dimension. 3-D *Pieces* will thus assemble to form 3-D *Targets*, while 1-D *Shapes* are simply straight line segments (each of which could alternatively be represented by a single number). The *Magic Target* will then be another straight line (equal in length to the sum of those numbers). From this is seen that **Numerical Magic Squares are in fact a particular kind of Geomagic Squares**.

2. MAGIC SQUARES : GENERAL CLASSIFICATION

The *Magic Squares* can be categorized, according to special properties they may have, in different ways.

When the *Cells* of the square are filled with *Numbers*, *Letters*/*Characters* or *Geometric Shapes*, the square is called, respectively, **Numeric Square**, **Letters/Words/Characters Square** and **Geomagic Square** (after Lee Sallows).

One of the most frequent classifications divides them in three classes: *Simple*, *Associated* and *Pandiagonal*. However, removing or adding them some properties, we can also consider in addition the following three: *Semi-Magic*, *Semi-Pandiagonal* and *Most Perfect*.

When they show special characteristics, take others names as: **Bimagic**, **Bordered** or **Concentric**, **Alfamagic**, **Latino**, **Domino**, **IXOHOXI**, **Prime**, **Serrated**, etc.

The previous six classes are defined, for *Normal Magic Squares of Order n,* in the following paragraphs 2.01 to 2.06, according the increasing complexity of their properties.

2.01 **Semi-Magic Square** – An array of n^2 *Cells* (*n Rows* x *n Columns*) where the sum of each *Row* and of each *Column* is equal to the *Magic Sum S*. The sum of one or the two *Main Diagonals* is different of *S* and hence the name *Semi-Magic*.

2.02 **Simple, Normal, Numeric Magic Square** of **Order n** or simply **Magic Square** – Array of n^2 *Cells* (*n Rows* x *n Columns*) each one filled with a certain number in a way that the sum of each *Row*, each *Column* and the two *Main Diagonals* all have the same value S, called *Magic Sum.*
The fulfillment of the above mentioned properties is the minimum requirement to qualify as a *Magic Square.*

2.03 **Associated** or **Regular** or **Symmetrical Magic Square** – *Magic Square* where the sum of all *Symmetric Pairs* is equal to the sum of the first and last terms of the series or $1 + n^2$.
In an *Odd Order Associated Magic Square*, the *Center Cell* is always equal to the middle number of the series i.e. $(1+n^2) / 2$.
No *Singly-Even Order Associated Magic Square* exists.

2.04 **Semi-Pandiagonal** or **Semi-Diabolic** or **Semi-Nasik Magic Square** – *Magic Square with* the following properties:
Even Order

- The sum of an *Opposite Short Diagonal Pair* with *n Cells*, is equal to the *Magic Sum*, *S*.

Odd Order

- The sum an *Opposite Short Diagonal Pair* with *n*-1 *Cells* plus the *Center Cell,* is equal to the *Magic Sum*, *S*;
- The sum of an *Opposite Short Diagonal Pair* with *n*+1 *Cells* minus the *Center Cell*, is equal to the *Magic Sum*, *S*

For a *Magic Square* of *Order n*, the *Magic Sum S*, is always calculated taken into account the *n Cells* of the *Rows*, *Columns* and *Pandiagonal Diagonals.* This is the reason why it is necessary to add or subtract the *Center Square Cell* to the *Opposite Short Diagonal Pair Cells*, when these are, respectively, *n-1* and *n+1*.

2.05 **Pandiagonal** or **Diabolic** or **Nasik** or **Continuous Magic Square** - A *Magic Square* where each *Broken Diagonal Pair* sum is equal to the *Magic Sum*, *S*.
The *Pandiagonal Magic Square* is considered one of the most sophisticated among the classes of Magic Squares;
No *Singly-Even Order Pandiagonal Magic Square* exists.

2.06 **Most Perfect Magic Square** - *Pandiagonal Magic Square* of *Doubly-Even Order*, with the two additional properties:

1. The *Cells* of any square of *Order 2*, (2 x 2 *Cells)* extracted from it, including *Wrap-Around*, sum the same value, $2\ (1+n^2)$;
2. Along the *Main* or *Broken Diagonals* any two members separated by *n*/2 *Cells*, are a *Complementary Pair*, i.e. sum $1+n^2$.

All the *Pandiagonal Squares* of *Order* 4 are *Most Perfect*. However, when *n*>4, the proportion Pandiagonal/Most Perfect decreases as *n* increases.

3. THE ISSUE: SOUVENIR SHEET, MINI SHEET, FIRST DAY COVER AND STAMPS

3.1 Souvenir Sheet B152 (1/1): Luo Shu Magic Square

The history of Chinese civilization is full of myths, legends and folk based on mythological beings. Among the first legendary Semi-divine cultural heroes, **Fu Xi**, **Shen Nong** (God of Agriculture) and **Huang Di** or **Yellow Emperor**, known as "**The Three Divine Emperors**", are the most venerated. After *Huang Di*, followed the "**Three Sage Kings**" **Yao**, **Shun** and **Yu**, "**The Great**", founder of Xia Dynasty. It was during the rein of *Yu*, "*The Great*", that many efforts were put into controlling the effects of great floods.

Among them, "*Divine Emperor Fu Xi*" and "*Sage King Yu*", were, respectively, witnesses of the visits of two mythical creatures: a "**Dragon-Horse**" and a "**Turtle**" showing different **Dot Patterns** on their backs.

Fu Xi, according to the legend, taught his subjects how to fish with nets, to hunt, to domesticate animals and to cook. One day, while he was standing on the banks of **Huang He** or **Yellow River** a creature with the form of a "*Dragon-Horse*" emerged from the river with a **Diagram** on its back, composed of **55 Dots**, in **5 sets**. Before submerging, it also left its foot print with **8 Patterns** composed of **Line Segments**. The *Diagram* became known as **He Tu** or **River Map** and the foot print **Ba Gua** or **Eight Trigrams**. The *Eight Trigrams* were later rearranged by King **Wen**, founder of Zhou Dynasty, which gave origin to the **Yi Jing 64 Hexagrams**.

Sage King Yu, "*The Great*", became legendary ruler for his introduction of a system of flood control, through the construction of dikes and canals. One day, when he was standing on the banks of **Luo River**, a tributary of the *Huang He* or *Yellow River*, a *"Turtle"* emerged from the water with a **Quadrangular Diagram** on its shells made of 9 small quadrangular contours with a series of dots inside each one, representing numbers from 1 to 9. *King Yu* was very surprised to discover that each *Row*, *Column* and *Main Diagonal* of the *Quadrangular Diagram* contained **15 Dots**. The *Quadrangular Diagram* became know as **Luo Shu** or **Luo River Scroll**. It is also commonly called **Jiu Gong Shu** or **Nine Halls Diagram**.

The *He Tu* (*River Map*) and *Luo Shu* (*Luo River Scroll*) are fundamental fabrics in the development of traditional Chinese culture extending its influence to religion, sociology, politics, philosophy, mathematics, medicine, civil engineering, etc.

3.2 **Mini-Sheet**

The **Mini Sheet** was conceived to present a disposition for the facial values (1 to 9 "patacas") equal to the disposition that the numbers 1 to 9 occupy in the **Luo Shu Magic Square**.

In this issue only 6 stamps are issued, corresponding to the first two *Rows*, with the third *Row* (presently without the corresponding 3 stamps) to be issued in the future.

With the actual issue and *Mini Sheet* design, the Macao Philately does not only intend to continue to divulgate and promote scientific knowledge, but also to present to philatelists and *Magic Squares* enthusiasts a unique piece that has never been produced in the world.

Besides, in the *Mini Sheet* several characteristics can be noticed:

- The use of **two different colours** (**black** and **red**) for the **odd** and **even facial values** of the stamps, when normally only one colour is indifferently used;
- The filling of the margins with the **12 Dudeney Patterns** and frequency occurrence of each one of them in the set of **880 different** *Magic Squares* that it is possible to construct for a *Natural Magic Square of Order 4*.

3.3 **First Day Cover: John R. Hendricks – Inlaid Magic Squares**

An **Inlaid Magic Square** is a *Magic Square* that contains within itself other **Lower Order Magic Squares**. The *Lower Order Inlaid Magic Squares* can be formed by any number inside (unlike a **Bordered Magic Square**, where the border must contain the lowest and the highest numbers in the series). They can also contain other *Inlaid Magic Squares* within themselves.

The First Day Cover shows an **Inlaid Magic Square of Order 9** with three *Inlaid Magic Squares of Orders 7, 5* and *3*. Note that the *Inlaid Magic Square of Order 3* is rotated 45 degrees and is also referred as an **Inlaid Diamond Magic Square**. The numbers used in the *Inlaid Magic Square of Order 9* are from 1 to 81, therefore it is a **Pure Magic Square**. The **Magic Sums** of the *Inlaid Magic Square of Order 9* and its *Lower Order Inlays* are: S_9=369, S_7=287, S_5=205 and S_3=123.

3.4 Stamp S172 (6/1): Dürer-Melencolia I

Albrecht Dürer, son of a goldsmith, was born in 1471, in Nuremberg, Germany. He became famous as painter, engraver, printmaker, mathematician and academic. He started as an apprentice to Michael Wolgemut when he was young. Later he had been in contact with famous artists like the Schongauer’s brothers, the goldsmiths Caspar and Paul, the painter Ludwig and the sculptor Nikolaus Gerhaert.

Nuremberg was not far away from Venice and Dürer went to Italy twice to study more advance techniques and new artistic expressions. During all these years he could transmit a strong influence and acquire a solid reputation that made him to be regarded as the greatest artist of Northern Renaissance.

After returning to Nuremberg for the second time, he created some famous artistic works as: the paintings, **Adam and Eve** (1507), **The Martyrdom of the Ten Thousand** (1508), **The Virgin with the Iris** (1508), the woodcuts, such as **The First Apocalypses Series** (1498), **The Great Passion** and **The Life of the Virgin** (1511), **The Second Apocalypses Series** (1511), and the well known “**Master Prints**” (Meisterstiche) **The Knight**, **The Death and The Devil** (1513), **Saint Jerome In His Study and Melencolia I** (1514).

Melencolia I is an engraving that includes in the upper right corner, under the bell, a *Normal Associated Magic Square* of *Doubly-Order*.

- The two middle *Cells* of the bottom *Row* show the date of the engraving, 1514.
- The *Magic Sum* is $S=4(4^2+1)/2=34$. In addition to the *Rows, Columns* and *Main Diagonals,* the sum *S* is also possible to be obtained in different ways as follows:
  - The four 2x2 Quadrants, e.g., 16+3+5+10=34

- The Central Square, e.g., 10+11+6+7=34
- The Corners of the four 3x3 Squares, e.g., 16+2+9+7=34
- The Corners of the centered 4x2 and 2x4 Rectangles, e.g., 3+2+15+14=34 and 5+8+9+12=34
- The Corners of the two diagonal 2x3 Rectangles, e.g., 2+8+15+9=34 and 5+3+12+14=34
- The two Skewed Squares, e.g., 8+14+9+3=34 and 2+12+15+5=34
- The Latin Cross Shapes, e.g., 3+5+15+11=34 and 2+10+14+8=34
- The Upside-down Cruciform (St. Peter's Cross) Shapes 3+9+15+7=34 and 2+6+14+12=34

• Any Pair of Cells that are symmetric around the Center sum 17.

3.5 **Stamp S172 (6/2): La Loubère** or **Siamese Construction Method**

There are several general methodologies to construct *Magic Squares* depending on the class (*Simple*, *Associated*, *Pandiagonal*, etc.) and *Order*. However these general methodologies not always apply for all the *Orders* of a certain class, as it is the case for the smallest Orders (3 and 4) because they are special cases. Through the times several methods for constructing Magic Squares have been created namely the following: **La Loubère** or **Siamese** – **Bachet de Méziriac** – **Philippe de la Hire** – **John Lee Fults** – **Ralph Strachey** – **Stairstep** – **Diagonal** – **Knight's Move** – **Lozenge** (John Conway) – **Dürer**, etc.

La Loubère methodology was created by Simon de la Loubère (1693), a French mathematician that learned it as ambassador to Siam, reason why it is also known by *Siamese*.

La Loubère method is one of the most popular to create *Magic Squares* of *Odd Order*. The main characteristic of this method consists in filling the *Cells* of the *Diagonals* in sequential order and moving upward and to the right.

Let's see how it works:

1. First, the middle *Cell* of the *Row*, is filled with number 1;
2. Whenever you reach the top side of the *Square*, move to the bottom *Cell* of the right *Column* and continue to fill the *Diagonal* upward to the right with the numbers in sequential order;
3. Whenever you reach the right side of the *Square*, move to the most left *Cell* of the upper *Row* and continue to fill the *Diagonal* as before;
4. Whenever you reach a *Cell* that is already filled move down one *Cell* and continue to fill the *Diagonal* as before;
5. If you reach to upper right corner *Cell* also move down one *Cell* and continue as 3.

In this stamp the lines over the *Cells* show the numeric sequence for filling the *Cells* according the methodology mentioned at paragraphs 1 to 5.

3.6 **Stamp S172 (6/3): Sator Palindrome**

The **Sator Square** or **Rotas Square** is a *Letters/Words Magic Square* that is composed of a *Latin Palindrome* with the five words – **SATOR AREPO TENET OPERA ROTAS** - that can be read forwards, backwards, upwards and downwards.

The oldest inscription was found in the ruins of Pompeii, which was destroyed in 79 A.D. by Vesuvius eruption of lava and ashes. Others were posteriorly found namely at Corinium (modern Cirencester in England) and Dura-Europos (in modern Syria). There is also a *Sator Square* in the museum at Conimbriga (near Coimbra in Portugal).

The correct translation and its meaning have been under dispute and speculation until the present. A word by word translation can be as follows:
Sator – Sower, seeder, planter, founder, progenitor, originator;
Arepo – Without a clear meaning, probably a proper name (Arepo);
Tenet – To hold, to keep, to possess, to master;
Opera – Work, care, aid, effort, service;
Rotas – Wheel, rotate.

As a sentence, dozens of translations were proposed e.g.:

- "The sower Arepo holds the wheels with effort";
- " The farmer Arepo keeps the world rolling";
- "Arepo the farmer holds the works in motion";
- "The Creator (or Saviour) holds the working of the spheres in his hands"

Some investigators have also speculated that if the five words are properly rearranged, a Greek Cross can be made, that reads horizontally and vertically **PATERNOSTER**, with the remaining letters (**A,A** and **O,O)** distributed by each of the 4 quadrants. This translates "**OUR FATHER, OUR FATHER**" with the letter *A* and *O* representing the **Alpha** and **Omega** – the **Beginning** and the **End**. This could make, as yet the speculation goes, the *Sator Magic Square* a safe and hidden way for the early Christians to identify themselves and signal their beliefs to each other without the danger of persecution.

3.7 **Stamp S172 (6/4): Franklin – Bent Diagonals**

Benjamin Franklin was born in Boston, Massachusetts, 17th January, 1706 and was one of the most influential “**Founding Fathers**” of the United States, earning the title of “**The First American**” for his fight for independence. He was also a man of many interests and talents.

In early days he worked as a printer, to become, with the course of the years, a **polymath**, **author**, **politician**, **scientist**, **inventor**, **musician**, **social activist**, **postmaster general**, **statesman** and **diplomat**.

As a *Postmaster* he was named in 1775 the First United States *Postmaster General*, establishing a postal system that was the basis for the present United States Post Office.

As an *Author* he started to publish the famous **Poor Richard's Almanack** that became at the time very popular reading. Some of the adages there published remain commons citations even at present.

As an *Inventor* and *Scientist*, among many of his inventions, there are: the **Bifocal Glasses**, the **Lighting Rod**, the **Flexible Urinary Catheter**, the **Glass Harmonica** etc. He also published several studies about Demography, Atlantic Ocean Currents, Electricity, Meteorology, Cooling Concept, etc.

Being a man with a strong character and clear ethical values, he established for himself, yet very young, as a guide, the following **13 virtues** that he continued to follow during his life: **Temperance**, **Silence**, **Order**, **Resolution**, **Frugality**, **Industry**, **Sincerity**, **Justice**, **Moderation**, **Cleanliness**, **Tranquility**, **Chastity** and **Humility**.

In addition to the numerous achievements, Benjamin Franklin also left his name associated to the “*Magic Squares*”. The “*Magic Squares*” of Benjamin Franklin represented in the stamps shows the same sum for the *Rows* and *Columns* but not for the *Main Diagonals,* i.e, it is only a *Semi Magic Square*. However, it possesses other magic properties as those associated with *Bent Diagonals* either *Continuous* or *Wrap-Around* with sum 260.

In the stamp, several **Bent-Up-Rows Diagonals** can be seen in different colours, including **Bent Wrap-Around Diagonals**.

3.8 Stamp S172 (6/5): Su Hui - Xuan Ji Tu - Palindrome

Su Hui (351 A.D.- ?) was a Chinese poetess that lived in **Former Qin** of the **Sixteen Kingdoms** period. She married **Dou Tao**, a government official who later was sent to defend the northern borders. Far away from her husband, she found out that he had taken a concubine. To console her unhappiness and try to bring him back she composed her **Palindrome Poem**, **Xuan Ji Tu**, an array of 29 *Lines* x 29 *Columns*, with **841 characters**, that can be read at least in **2848 different ways**, namely, forward, backward, horizontally, vertically and diagonally. After reading the poem, *Dou Tao* left his concubine and return to *Su Hui*, and the love between them became very strong and forever.

This stamp is a square only with 15 *Lines* x 15 *Columns* extracted from the central part of the 29 *Lines* x 29 *Columns* square that constitutes the full poem *Xuan Ji Tu.* Once the poem can be read in so many different ways, for easier understanding on how it can be read, it is necessary to identify some sets of characters, namely:

- The **Internal Red Frame**, i.e., the **Central Red Square** (3x3), without the character **心 (xīn).** It is said that this character did not appear in the original poem and was added later by another scholar;
- The **Black Frame** evolving around the *Central Red Square*;
- The **4 Black Squares** (4x4) at the inner corners of the **Peripheral Red Frame**;
- The **4 Blue Rectangles** (5x4) between the *Black Squares;*
- The **Peripheral Red Frame**;
- The **Diagonals**.

Si Ku Quan Shu (**The Imperial Collection of Four**) and **Shi Yuan Zhen Pin:Xuan Ji Tu** by **Li Wei** are references used for explaining on how to read the poem in different ways.

1. The **Internal Red Frame**, 8 characters.
   Starting from the middle character **詩(shī)** and read it in anti-clockwise with 4 characters for each sentence, we will get two sentences "詩圖璣琁，始平蘇氏。**(**shī tú jī xuán，shǐ píng sū shì。)**"**. Starting from the character **蘇(sū)** to read in the same way, we will get another two sentences "蘇氏詩圖，璣琁始平。" (sū shì shī tú , jī xuán shǐ píng).
   Finally, we can get "詩圖璣琁，始平蘇氏。蘇氏詩圖，璣琁始平。" with the meaning of "*Xuan Ji Tu* is composed by *Su Hui* who lived in Shi Ping County(始平縣) and it is the origin of *Palindrome Poem.*"

2. The **Black Frame**, 16 characters.
   It includes 16 characters in black colour. Starting from the right lower corner **怨義(yuàn yì)** and read in clockwise with 4 characters for each sentence, we get "怨義興理，辭麗作比，端無終始，詩情明顯。 (yuàn yì xīng lǐ，cí lì zuò bǐ，duān wú zhōng shǐ，shī qíng míng xiǎn。)" with the meaning of "I use beautiful words and phrases weaved in this brocade to express my ethical complaints and the rationales of them. However, my love for you continues where you can understand it from the deep emotion meanings embedded in this poem".

Starting from the left upper corner **端無(duān wú)** and read in clockwise with 5 characters for each sentence, in which the fifth character of last sentence will be repeated as the first character of next sentence. Hence we can get“端無終始詩，詩情明顯怨。怨義興理辭，辭麗作比端。(duān wú zhōng shǐ shī，shī qíng míng xiǎn yuàn。yuàn yì xīng lǐ cí，cí lì zuò bǐ duān。)” with the meaning of “I use this poem to express my love to you. The poem contains my obvious discontentment as well. I need to present in the text the rationales behind my ethical complaints. It is because of my love to you that I am using beautiful words and phrases to write this poem.”

Different poems can be extracted starting from different corners and read in 4-characters, 5-characters, clockwise or anti-clockwise etc. It is said that at least 24 poems can be read.

3. The **4 Black Squares** (4x4).
   Each square contains 16 characters. Starting from the right upper corner **思感(sī gǎn)** and read it in zigzag way, 4 characters for each sentence. We can get“思感自寧，孜孜傷情。時在君側，夢想勞形。(sī gǎn zì níng，zī zī shāng qíng。shí zài jūn cè，mèng xiǎng láo xíng。” with the meaning of “Thinking of you and the time we passed make me sad and restless. I cannot sleep because I am missing you, and this makes me thin and pallid”.
   The characters of the *Black Squares* of characters can be read in *Row* by *Row*, *Column* by *Column*, zigzag in clockwise or anti-clockwise. It is said that at least 176, 4-character poems can be read.

4. The **4 Blue Rectangles** (5x4) between the *Black Squares.*
   Each square contains 20 characters. Take the right rectangle as example and starting from the upper right corner **寒歲(hán suì)**, read in zigzag way with 5 characters for each sentence, we can get “寒歲識凋松，始終知物貞。顏喪改華容，士行別賢仁。(hán suì shí diāo sōng，shǐ zhōng zhī wù zhēn。 yán sāng gǎi huá róng，shì xíng bié xián rén。)” It means “Pine trees stand firmly in the cold winter. Since you left me, my face grows aging; however, my love for you is eternal just like the pine trees.”

   The characters of the *Blue Rectangles* can be read in *Row* by *Row*, *Column* by *Column*, zigzag, in clockwise or anti-clockwise. It is said that at least 176, 5-character poems can be read.

5. The **Peripheral Red Frame**, 56 characters.
   A frame of 56 characters with 8 special **Rhyme Characters**“欽、林、麟、身、溱、沈、神、殷 (qīn、 lín、lín、shēn、shēn、chén、shén、yīn)” arranged in corners and mid-point of each side.

For example, starting from upper right corner 欽**(qīn)** and read it clockwise till the lower left corner 沙**(shā)** with 7 characters for each sentence, we can get"欽岑幽巖峻嵯峨，深淵重涯經網羅。林陽潛曜翳英華，沈浮異逝頹流沙。(qīn cén yōu yán jùn cuó é，shēn yuān chóng yá jīng wǎng luó。lín yáng qián yào yì yīng huá，chéng fú yì shì tuí liú shā。)" with the meaning of "The long curve bank of river, the danger ridge of high mountain, the deep of dark pond, make me fear. I feel depress because my letter to you is lost, just like the warm sunlight for beautiful flowers is blocked by dense forests".

Starting from different *Rhyme Characters* and read in different way, it is said that at least 96 7-character poems can be read.

6. The **Diagonals**, 29 characters each.
   There are two *Main Diagonals* in the brocade. Starting from the 嗟**(jiē)** near the upper right corner, and read it diagonally to the lower left corner with 7 characters for each sentences, we can get"嗟中君容曜多欽，思傷君夢詩璇心。氏辭懷感戚知麟，神輕粲散哀春親。(jiē zhōng jūn róng yào duō qīn， sī shāng jūn mèng shī xuán xīn。shì cí huái gǎn qī zhī lín，shén qīng càn sǎn āi chūn qīn。)" With the meaning of "Thinking of you make me pale in face, I can only express my love in my poem and meet you in my dream. Although Spring comes, I am still in low spirit and feel sad." Starting from different corner and read in different way, it is said that at least 96 7-characters can be read.

   Su Hui used a lot of *Rhyme Characters* which are ingenious arranged in the *Xuan Ji Tu*, and because of this arrangement, even when we start from different character and read in different ways, we still can extract a meaningful poem.

3.9 Stamp S172 (6/6): Lee Sallows – Panmagic 3x3

Born in England in 1944, as a boy *Lee Sallows* became interested in short wave radio, after which he was to find work as a technician in various branches of the electronics industry. In 1970 he moved to Nijmegen in the Netherlands where he was employed by the Radboud University as an electronics engineer, until his retirement in 2009.

After developing an interest in recreational mathematics, he became an expert on the theory of *Magic Squares,* a topic to which he contributed several new variations, most notably *Alphamagic* and *Geomagic Squares*. **Sallows has an Erdős number of 2**.

Having become strangely attracted to a **formula due to Édouard Lucas** that characterizes the structure of every *3×3 Magic Square* (among them the *Luo Shu*), Sallows speculated that it might contain hidden potential.

This speculation was confirmed in 1977 when he published a paper that correlated every *Magic Square* of *Order 3* with a unique parallelogram on the complex plane. In an improbable move, he then tried substituting the variables in the Lucas formula with geometrical forms, an eccentric notion that led immediately to the invention of *Geomagic Squares*. It turned out to be an unexpected consequence of this find that **Traditional Magic Squares** using numbers **were now revealed as One-dimensional Geomagic Squares**. Other researchers have since then taken notice, among them Peter Cameron who has suggested that "an even deeper structure may lie hidden beyond *Geomagic Squares*".

The stamp is a *Pandiagonal* or *Nasik 2-D Magic Square* of Order 3, or one in which, in addition to *Rows* and *Columns*, all six *Diagonals* are *Magic*, including the 4 so-called *Broken Diagonals*. In this case the *Magic Target* can also be formed by any three of the four corner *Pieces*. This square is of interest because a *Numerical* equivalent is impossible to construct. The possibility of finding a *Geometrical 3×3 Nasik Square* was thus anything but certain, and their initial discovery an event to celebrate. The resort to disjoint *Pieces* (all of them **Pentominoes**) is an indication of the difficulty encountered in finding it.

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Science And Technology – Magic Squares II

Introductory Note: The **Magic Squares** series consists of two issues with 9 stamps. The first issue was published on 9 October 2014. In this issue the last 3 stamps with the facial values of 8, 1 and 6 patacas are published.

The theme of *Magic Squares* is transversal in Chinese and Western cultures, **Macao Post** intends to publicize and promote the **scientific and cultural aspects** of this theme as well as to create a **unique product** in the history of Philately.

Due to limited space in the **Information Brochures**, additional explanations of the **Souvenir Sheets**, **Sheetlets**, **First Day Covers** and **Stamps**, as well as some technical terms used to characterize and define *Magic Squares* can be found at the following **Site** of *Macao Post*:

- **Magic Squares Issues I** and **II –** http://goo.gl/lOHEqo.

Souvenir Sheet: Method of Knight's Tour

There are several general methodologies to construct *Magic Squares* depending on the **Class** and **Order**. Among them, the following can be mentioned: **La Loubère** or **Siamese**, **Bachet de Méziriac**, **Philippe de la Hire**, **John Lee Fults**, **Ralph Strachey**, **Knight's Tour**, **Dürer**, etc.

In the **Souvenir Sheet** of this issue, the **Method of Knight's Tour** is used to construct a **Magic Square of Order 16** with a **Closed** or **Reentrant Tour**.

This method consists, starting in an **Initial Cell**, in which the number 1 is allocated, to fill the *Cells* numerically and sequentially from 1 to n^2, of a **Square of Order n**, using the characteristic movements of a *Knight Jump*, as in the Chess game.

Once the **Tour** is established, between the **Initial Starting Cell** and the **Final Arriving Cell**, if it is possible to jump from the *Final Arriving Cell* to the *Initial Starting Cell* with a legal *Knight movement*, the *Tour* is called **Closed** or **Reentrant** and, in this case, the *Initial Starting Cell* can be anyone. On the contrary, the *Tour* is called **Open** or **Non-reentrant**.

The interest aroused by the creation of *Magic Squares* using the *Method of Knight's Tour* in different dimension *Boards*, led to studies that concluded not to be possible to exist a *Magic Square Tour* in $n \times n$ *Boards* with *n Odd*. However, it is possible for *Boards* of **Order 4 k×4 k**, with **k>2**.

The *Magic Square of Order 16* with a *Closed Tour* in the *Souvenir Sheet* of this issue was published by the author **Joseph S. Madachy**, in 1979.

As shown in the *Souvenir Sheet*, the allocation of the numbers 1 to 256 in the *Cells* is sequential and complies with the *Knight Jump* rule of Chess game. The **Magic Sum** is 2056.

Sheetlet

The **Sheetlet** presents a disposition for the face values of the stamps (1 to 9 patacas) equal to the disposition that the numbers 1 to 9 occupy in the **Luo Shu Magic Square**.

In this issue, the last three stamps with the **face values** of **8**, **1** and **6** patacas, corresponding to the **Inferior Row** as mentioned in the *Introductory Note* are issued.

On the **Lateral Margins**, two *Magic Squares* **Tiling Schemes** are presented, proposed by **David Harper** which correspond to the **binary** and **decimal numerical bases**.

First Day Cover: Yang Hui Magic Circles

The XIII Century was probably one of the most important periods in the History of Chinese Mathematics, with the publication of **Shu Shu Jiu Zhang** (**數書九章**), 1247, by **Qin Jiu Shao** (**秦九韶**) and **Ce Yuan Hai Jing** (**測圓海鏡**), by **Li Ye** (**李冶**), followed 15 years later, by the works of **Yang Hui** (**楊輝**).

Yang Hui (1238-1298), a Chinese mathematician born in Qiantang (錢塘) (modern Hangzhou (杭州)), Zhejiang Province (浙江省), during the late Song Dynasty (宋朝) (960-1279). His best known work was **Yang Hui Suanfa (楊輝算法)**, **Yang Hui's Methods of Computation,** which was composed of 7 volumes and published in 1378.

The topics covered by *Yang Hui* include Multiplication, Division, Root-extraction, Quadratic and System Equations, Series, Computations of Areas of Polygons as well as *Magic Squares*, *Magic Circles*, the **Binomial Theorem** and, the best known work, his contribution to the **Yang Hui's Triangle**, which was later rediscovered by **Blaise Pascal**, 1653.

The Bottom Left Corner of the *First Day Cover* presents the **Yang Hui Magic Circles**. These **Nine Circles** are composed by **72 Numbers**, from 1 to 72, with each individual *Circle* having **8 Numbers**. The **Neighboring Numbers** make **Four Additional Circles**, each with *8 Numbers,* the total **Sum** *of the 72 Numbers* is **2628** and the **Sum** *of the 8 Numbers* **in each Circle** is **292**.

Stamp (3/3) : Inder Taneja – IXOHOXI 88

Inder Taneja, Professor of the Department of Mathematics of University of Santa Catarina, Brazil, from 1978 to 2012. He has published more than 100 research papers in internationally renowned journals.

IXOHOXI Magic Squares are a special series that not only show common properties like other *Magic Squares*, as well as being **Pandiagonals**, but also include alternative properties such as **Symmetries**, **Rotations** and **Reflections**.

The word *IXOHOXI* is itself a **Palindrome** and *Symmetric* (*Reflection*), in relation to its center "H". As the 10 digits (0 to 9) use the number style of a **7 Segments LED Display**, in which only 5 digits (0, 1, 2, 5 and 8) remain the same after a **180 Degrees Rotation**. It should be noted that the 4 digits (**0**, **1**, **2** and **5**) used to construct the **Magic Square of Order 4**, are precisely the same digits that constituted the year **2015**, year of its publication as a stamp.

Taking into consideration the 5 digits and their *Symmetric Properties*, *Inder Taneja* created the *IXOHOXI Universal 88 Magic Square,* reproduced in this stamp, has the following properties:

The *Magic Square* still remains a *Magic Square*:

- After a **Rotation of 180 Degrees**;
- After **changing the order of the digits** in the *Cell* numbers, i.e. 82 to 28;
- If it is **seen in a mirror**, or **reflected in water** or **seen from the back** of the sheet;
- The *Magic Sum S* of the *Magic Square of Order 4* is equal to **88**, number that also enjoys *Symmetrical* properties.

Stamp (3/1): McClintock / Ollerenshaw – Most Perfect

A **Most-Perfect Magic Square** is a **Pandiagonal Magic Square of Doubly Even Order –** with additional two proprieties:

- The *Cells* of any square of **Order 2**, (**2×2 Cells**) extracted from it, including **Wrap-Around**, sum up to the same constant value, $\mathbf{2(1+n^2)}$;

- Along the **Main** or **Broken Diagonals**, any two numbers separated by **n/2 Cells**, are a **Complementary Pair**, i.e. sum $\mathbf{1+n^2}$.

In the case of the *Most-Perfect Magic Square of Order 8* reproduced in the stamp, the mentioned properties show the following results:

- $2(1+n^2) = 2(1+8^2) = 130$ E.g.: (59+38+7+26) = (48+33+18+31) = 130
- $(1+n^2) = (1+8^2) = 65$ E.g.: (1+64)=(34+31) = (25+40) = 65

All the *Pandiagonal Squares of Order 4* are *Most-Perfect*. However, when *n>4* the proportion *Pandiagnal to Most-Perfect* decreases as *n* increases.

It is not possible to establish the history of *Most-Perfect Magic Squares* without to mention **Kathleen Timpson Ollerenshaw**. In 1982, with **Hermann Bondi**, she developed a **mathematical analytical construction** that could verify the number **880** for the **essentially different** *Magic Squares of Order 4*. After this achievement she began to study *Pandiagonal Magic Squares* based on works published by **Emory McClintock** in 1897. After several years, in 1986, *Kathleen Ollerenshaw* published a paper where, making use of *Symmetries*, she proved that there are **368640** *essential different Most-Perfect Magic Squares of Order 8*.

Step by step, finally she could discover how to construct and how to count the total number of *Most-Perfect Magic Squares* of all with an **Order Multiple of 4**.

Together with **David Brée**, who helps her organize her research notes and proof-reading, they finally published the book "**Most-Perfect Pandiagonal Magic Squares: Their Construction and Enumeration**" in 1998.

Stamp (3/2): David Collison – Patchwork

David M. Collison (1937-1991) was born in United Kingdom and lived in Anaheim, California. He was a fruitful creator of *Magic Squares* and **Cubes**. He specialized in **Generalized Shapes** from which he created the **Patchwork Magic Squares**.

A *Patchwork Magic Square* is an **Inlaid Magic Square –** one *Magic Square* that contains within it other *Magic Squares* or **Odd Magic Shapes**. The most common *Shape* is **Magic Rectangle**, but **Diamond**, **Cross**, **Elbow** and **L** *Shapes* can also be found.

These *Shapes* are *Magic* if the *Sum* in each **Direction** is proportional to the number of *Cells*. The **Patchwork Magic Square of Order 14** reproduced in this stamp has the following proprieties:

- Contain: Four **Order 4 Magic Squares**, *4×4*, in the **Quadrants**; One **Magic Cross**, *6×6*, in the **Centre**; Four **Magic Tees**, *6×4*, on the **Centre Sides**; Four **Magic Elbows**, *4×4*, in the **Corners**.
- All the *Shapes* sum to a **Constant** which is directly proportional to the number of *Cells* in a *Row*, *Column* or *Diagonal*: $\mathbf{S_2=197}$; $\mathbf{S_4=394}$; $\mathbf{S_6=591}$; $\mathbf{S_{14}=1379}$.

Bibliography: Available at the website of *Macao Post* mentioned above.

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